NEW METHOD FOR ESTIMATING THE AVERAGE S-WAVE VELOCITY OF THE GROUND

Katsuaki KONNO* and Shun’ichi KATAOKA**

ABSTRACT

There is a good correlation between the time-weighted average S-wave velocity from surface to a depth of 30 m (Vs30) and the amplification factor for peak velocity. The Vs30 is usually estimated from PS logging. A newly proposed method for estimating Vs30 uses a phase velocity of the fundamental mode Rayleigh wave obtained from the array observation of microtremors. This method is based on the phenomenon that the Vs30 is approximately equal to phase velocity of Rayleigh wave at wavelengths of 35 to 40 m. In this paper, we compare the Vs30 and a phase velocity of Rayleigh wave at 85 sites around Tokyo. At these sites, velocity structure is explored by PS logging. Results from calculations show that the proposed method is applicable. Furthermore, we point out that the phase velocity at wavelengths of 35 to 40 m is easily obtained by a small array.

Introduction

Investigations based on the observation and analyses of seismic ground motion have revealed that the average S-wave velocity of ground surface to a certain depth shows strong correlation with the relative amplification (Midorikawa, 1987; Joyner and Fumal, 1984; Borcherdt et al., 1991). In this study, we investigate a newly proposed method for estimating the average S-wave velocity to a depth of 30 m (Vs30) defined as

\[
Vs30 = \frac{30}{\sum_{i=1}^{N} \frac{H_i}{V_{S_i}}} \quad (1a)
\]

\[
30 = \sum_{i=1}^{N} H_i \quad (1b)
\]

where \( H_i \) and \( V_{S_i} \) are the thickness and S-wave velocity of \( i \)-th layer, respectively.

* Assistant Professor, Shibaura Institute of Technology, 3-9-14, Shibaura, Minato-ku, Tokyo 108-8548 Japan
** Research Engineer, Institute of Technology, Shimizu Corporation, 3-4-17, Echujima, Koto-ku, Tokyo 135-8530 Japan
Midorikawa et al. (1994) investigated a relationship between $Vs30$ and amplification factors for peak velocity for 47 sites where S-wave velocity profiles are available during the 1987 Chiba-ken-toho-oki, Japan earthquake. As a result, they showed the following regression model:

\[
\log A = 1.83 - 0.66 \log Vs30 \pm 0.16
\]

\[
(100 < Vs30 < 1500)
\]

where $A$ is the amplification factor to Tertiary ground and the last term with '±' is standard deviation. Therefore, if we obtain $Vs30$ by any method, we can estimate the amplification factor by using Eq. 2. In the following sections, at first we show that the $Vs30$ is able to estimate from the phase velocity of Rayleigh wave at wavelengths of 35 to 40 m, then point out that we can estimate the phase velocity using a small array observation of microtremors.

**Relationship between S-wave Velocity structure and Phase Velocity of Rayleigh Wave**

First of all, we explain an approximate method (Konno and Kataoka, 1999) for estimating phase velocities of Rayleigh wave from S-wave velocity structure of the ground. Let consider the S-wave that is excited at the surface. At depth $D$, the travel time $tt$ is expressed as follows.

\[
 tt(D) = \int_0^D \frac{dz}{Vs(z)}
\]

where $Vs(z)$ is S-wave velocity at a depth of $z$. We assume that depth $D$ is equal to the wavelength of Rayleigh wave at a period of $T$ which is equal to the travel time $tt$. Then, We introduce an approximate phase velocity $C_k$ that is defined by eq.4. As shown in eq.4, the $C_k$ is a function of period $T$ and obtained from dividing depth $D$ by travel time $tt$.

\[
 C_k(T) = D / tt
\]

\[
 T = tt(D)
\]

We compare the phase velocity $C_k$ with the theoretical phase velocity of Rayleigh wave and Love wave using two ground models those are displayed in Fig.1. These ground models were obtained from PS logging around Tokyo. In Fig.2, the $C_k$ is shown with the theoretical phase velocity of Rayleigh wave and Love wave. The result shows that the phase velocity $C_k$ is very close to the phase velocity of Rayleigh wave but not close to the Love wave. It is interesting that the phase velocity $C_k$ approximately corresponds to that of Rayleigh wave rather than Love wave whose phase velocity depends on S-wave velocity and density structures. Using 85 ground models, we confirmed that the $C_k$ obtained by eq.4 is fairy good approximation of the phase velocity of Rayleigh wave.

**Estimation of Vs30 from Phase Velocity of Rayleigh Wave**
From a different point of view, $C_k$ in Eq. 4 corresponds to the average S-wave velocity to a depth $D$. If $C_k$ is always close to the theoretical phase velocity of Rayleigh wave, the $V_{S30}$ is estimated from phase velocity of Rayleigh wave at wavelength of 30 m as follows.

$$V_{S30} \approx C(30)$$

where $C$ as a function of wavelength is a theoretical phase velocity of Rayleigh wave. In practice, the phase velocities depend on not only S-wave velocity structure but also structures of P-wave velocity and density of the ground. Therefore, we investigate a relationship between $V_{S30}$ and phase velocity at other wavelengths in addition to wavelength of 30 m.

Fig. 3 shows location of 85 sites where soil profiles are used in this study. Fig. 4 shows a relationship between $V_{S30}$ and depths to basement at 85 sites. The range of $V_{S30}$ is from 80 to 390 m/s. Fig. 5 shows a relationship between P- and S-wave velocities. It is found that Poisson's ratio of almost all layers exceeds 0.45. Fig. 6 shows a relationship between $V_{S30}$ and theoretical phase velocity at wavelengths of 5 to 60 m by 5 m. We can see that $V_{S30}$ are approximately equal to phase velocities at wavelength of 35 m ($C_{35}$) and 40 m ($C_{40}$). If we obtain $C_{35}$ or $C_{40}$ by any way, we can regard these values as $V_{S30}$. We therefore propose a new empirical relation as follows.

$$V_{S30} \approx C_{35} \text{ or } C_{40}$$

**Range of Wavelength possible to estimate a Phase Velocity of Rayleigh Wave using Array Observation of Microtremors**

We can estimate the phase velocity of Rayleigh wave using array observation of microtremors. A range of the detectable wavelength of Rayleigh wave depends on array size. To the measurements, either the spatial autocorrelation (SPAC) method (Aki, 1957) or the frequency-wavenumber (F-K) method (Capon, 1969) could be applied. When the array is employed as shown in Fig. 7, Miyakoshi (1995) obtained the result of the range of wavelength possible to estimate the phase velocity form numerical and experimental studies as follows.

SPAC method: $R \leq \lambda \leq 10R$

F-K method: $\sqrt{3}R / 2 \leq \lambda \leq 5R$

where $R$ is a radius of the array in Fig. 7. In this case, if these two methods are applied to the same microtremors data, reliability of the results will increase. The overlapped range of two methods is

$$R \leq \lambda \leq 5R$$

where $\lambda$ is wavelength of Rayleigh wave. A gray area as shown in Fig. 8 represents the above-mentioned range. Thus, we can estimate the phase velocity at wavelengths of 35 to 40 m using an array observation with a radius of 7 to 40 m. This fact indicates that we can estimate
Vs30 at a site where there is an open space having a radius of about 7 to 40 m. It is not difficult to look out this size space even in a very crowded urban area.

Conclusions

We examined a relationship between Vs30 and theoretical phase velocities of Rayleigh wave at wavelengths of 5 to 60 m by 5 m at 85 sites. As a result, it is found that Vs30 are approximately equal to phase velocities at wavelengths of 35 and 40 m. We therefore proposed a new method that we can regard phase velocities of Rayleigh wave at wavelengths of 35 or 40 m as Vs30. The phase velocities at wavelengths of 35 to 40 m are obtained by an array observation of microtremors with a radius of 7 to 40 m. Thus, proposed method is applicable to a crowded urban area.

References

Figure 1. Soil profiles at two sites.

Figure 2. Approximate phase velocity and theoretical phase velocities of Rayleigh and Love waves at two sites.

Figure 3. Location of 85 sites where soil profiles are available.

Figure 4. The depth to basement versus Vs30 at 85 sites.

Figure 5. P-wave velocity versus S-wave velocity of 85 sites.
Figure 6. Phase velocity $C(\lambda)$ of Rayleigh wave at wavelength $\lambda$ versus $V_{s30}$.

Figure 7. Array shape used in this study.

Figure 8. Relationship between radius $R$ as shown in Fig.7 and detectible wavelength of Rayleigh wave.